

General Certificate of Education  
June 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Mechanics 4**

**MM04**

Wednesday 21 June 2006 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 Three forces  $2\mathbf{i}$ ,  $3\mathbf{i} - 5\mathbf{j} + a\mathbf{k}$  and  $b\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$  act at the points with coordinates  $(1, 1, 0)$ ,  $(0, 0, 0)$  and  $(-1, 2, 1)$  respectively, where  $a$  and  $b$  are constants.

Given that the three forces form a couple, find:

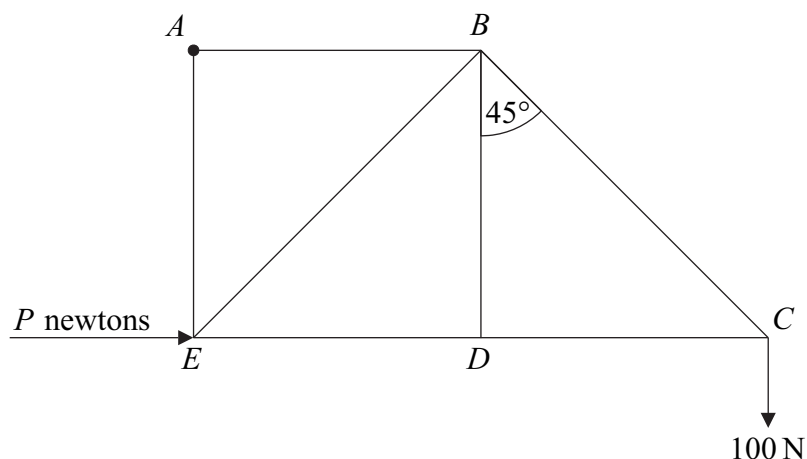
- (a) the values of  $a$  and  $b$ ; (2 marks)
- (b) the moment of the couple. (4 marks)

- 2 Forces  $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ ,  $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$  act at the points with coordinates  $(0, 0)$ ,  $(0, 3)$ ,  $(3, 4)$  and  $(4, 0)$  respectively.

- (a) (i) Find the magnitude of the resultant  $\mathbf{F}$  of this system of forces. (3 marks)
- (ii) Show that the line of action of  $\mathbf{F}$  cuts the  $y$ -axis at the point  $(0, 2)$ . (4 marks)
- (b) The system of forces is equivalent to a force acting at the origin together with a couple  $C$ . Write down the magnitude of  $C$  and indicate its sense on a diagram. (2 marks)

- 3 A framework is composed of seven light smoothly-jointed rods  $AB$ ,  $AE$ ,  $BE$ ,  $BD$ ,  $ED$ ,  $BC$  and  $DC$ , so that  $ABDE$  is a square and  $BDC$  is a right-angled triangle. The rod  $AB$  has length  $l$  and angle  $CBD = 45^\circ$ .

The framework is in a vertical plane and is freely hinged at  $A$  to a fixed support. A vertical force of  $100\text{ N}$  acts at  $C$ . The rod  $AE$  is kept vertical by a horizontal force of magnitude  $P$  newtons applied at  $E$ , as shown in the diagram.

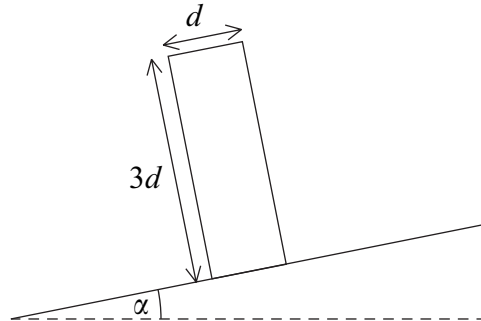


- (a) Show that  $P = 200$ . (2 marks)
- (b) (i) Find the magnitude of the reaction force on the framework at  $A$ . (2 marks)
- (ii) Find the angle between this reaction force and the horizontal, giving your answer to the nearest degree. (1 mark)
- (c) Find the magnitudes of the forces in each of the rods  $AB$ ,  $AE$  and  $BE$ , stating whether they are in tension or compression. (5 marks)

**Turn over for the next question**

**Turn over ►**

- 4 A uniform solid circular cylinder is in equilibrium with one plane face on a rough inclined plane. The plane is inclined to the horizontal at an angle  $\alpha$  degrees, which can be varied. The cylinder has weight  $W$ , diameter  $d$  and height  $3d$ .

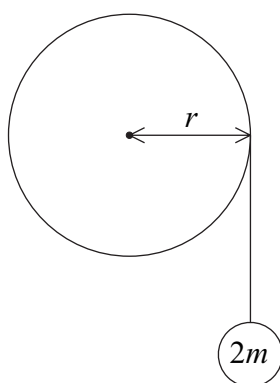


- (a) Draw a diagram showing the forces acting on the cylinder. (2 marks)
- (b) If the plane is sufficiently rough to prevent sliding, find the maximum value of  $\alpha$  for the cylinder to remain in equilibrium. (3 marks)
- (c) The coefficient of friction between the cylinder and the plane is  $\frac{2}{9}$ . If the value of  $\alpha$  is gradually increased from zero, show that the cylinder will slide before it topples. (5 marks)

- 5 A light inextensible string is wrapped several times around a uniform cylinder and a particle of mass  $2m$  is attached to the free end of the string.

The cylinder, of radius  $r$ , is free to rotate about a smooth fixed horizontal axis through its centre, perpendicular to its plane face. The moment of inertia of the cylinder about this axis is  $4mr^2$ .

The system is released from rest with the particle hanging freely. After time  $t$ , the cylinder has turned through an angle  $\theta$  radians. Assume that during this subsequent motion no slipping of the string occurs.

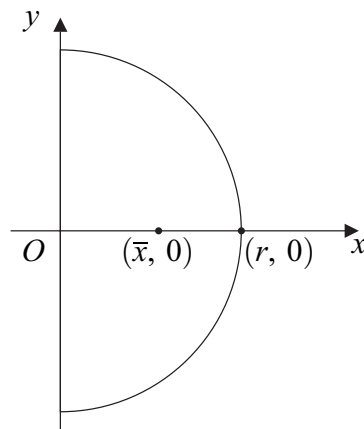


- (a) Show that  $\ddot{\theta} = \frac{g}{3r}$ . (6 marks)
- (b) Hence find an expression for the tension in the string in terms of  $m$  and  $g$ . (1 mark)

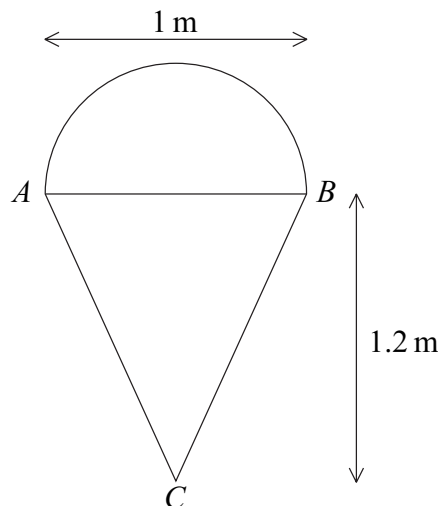
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- 6 (a) A uniform semicircular lamina of radius  $r$  has its centre at the origin and its axis of symmetry along  $Ox$ . The position of its centre of mass has coordinates  $(\bar{x}, 0)$ .



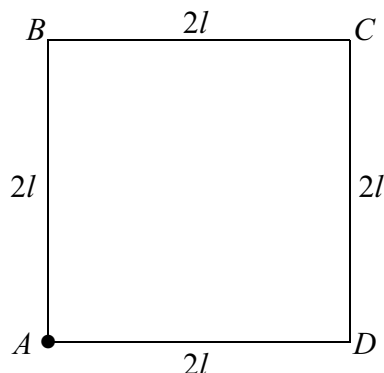
- (i) Show that  $\frac{1}{2}\pi r^2 \bar{x} = \int_0^r 2x\sqrt{r^2 - x^2} dx$ . (4 marks)
- (ii) Hence prove that  $\bar{x} = \frac{4r}{3\pi}$ . (3 marks)
- (b) The diagram below shows a uniform lamina, used as an advertising feature in a local supermarket to promote ice cream. It consists of a semicircle, of diameter  $AB = 1\text{ m}$ , and an isosceles triangle  $ABC$ , where  $C$  is at a distance  $1.2\text{ m}$  from  $AB$ .



- (i) State the distance of the centre of mass of the **triangle** from  $C$ . (1 mark)
- (ii) Show that the distance of the centre of mass of the **semicircle** from  $C$  is approximately  $1.41\text{ m}$ . (1 mark)
- (iii) Find the distance of the centre of mass of the complete lamina from  $C$ . (4 marks)
- (c) The lamina is freely suspended from  $A$ . Find the angle that  $AB$  makes with the vertical through  $A$ , giving your answer to the nearest degree. (3 marks)

- 7 A rigid square framework  $ABCD$  is formed from four identical uniform rods. Each rod has length  $2l$  and mass  $m$ .

The framework can rotate freely in a vertical plane about a horizontal axis through  $A$  perpendicular to the plane of the square  $ABCD$ .



- (a) Show that the moment of inertia of the **rod  $BC$**  about the axis is  $\frac{16ml^2}{3}$ . (4 marks)
- (b) Particles of masses  $4m$ ,  $3m$ ,  $2m$  and  $m$  are fixed at the vertices  $A$ ,  $B$ ,  $C$  and  $D$  respectively.

Show that the moment of inertia of the whole system about the axis through  $A$  is  $\frac{136ml^2}{3}$ . (6 marks)

- (c) The system is released from rest with  $AD$  horizontal and  $B$  vertically **above**  $A$ . Find, in terms of  $g$  and  $l$ , the angular velocity of the system when  $B$  is vertically **below**  $A$ . (7 marks)

**END OF QUESTIONS**

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